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OPTIMAL CONTROL OF GAS PIPELINES VIA INFINITE-DIMENSIONAL ANALYSIS

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SUMMARY

A general optimal control approach employing the principles of calculus of variations has been developed to determine the best operating strategies for keeping the outlet pressure of gas transmission pipelines around a predetermined value while achieving reasonable energy consumption. The method exploits analytical tools of optimal control theory. A set of partial differential equations characterizing the dynamics of gas flow through a pipeline is directly used. The necessary conditions to minimize the specific performance index come from the infinite-dimensional model. The optimization scheme has been tested on a pipeline subject to stepwise change in demand.

KEY WORDS: optimal control; optimization; gas pipelines

1. INTRODUCTION

Sources of global primary energy consumption are oil, coal and natural gas in the order of their share in total consumption. The use of natural gas, which occupies third rank, is increasing rapidly and it is widely believed that natural gas will replace coal then oil by the end of the first or second decade of the 21st century. This will obviously bring about an increase in natural gas flow flowing through pipelines (which is the principal system for transporting gas) from suppliers to consumers and the construction of new transmission and distribution systems.

The basic control element in a natural gas system is the compressor stations. The compressor stations in the transportation systems operate to compensate for pressure drop resulting from pipeline resistance. In practice, compressors are controlled on either outlet pressure or flow rate. While natural gas dispatchers are managing the system via compressors to keep it in the safe region, their first objective is to deliver gas to consumers at around a specified contract pressure. This is not an easy task, since true steady state flow (i.e. *in* equals *out*) rarely if ever exists in the real world. In actuality, customer usage is continually changing. Under this condition the flow rate is clearly unsteady and the control of this dynamic system requires much more rigorous perspective. The motivation of the present work is to investigate the optimal operating policies for the control of outlet pressure to accomplish the aforementioned objective. Minimizing the energy consumption is another objective.

The goal of this study is to apply optimal control theory to develop the best operating strategies for keeping the outlet pressure of gas transmission pipelines around a predetermined value while achieving *reasonable* energy consumption. In particular, the problem is to determine the control policy that

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extremizes a specific performance index subject to the constraints imposed by the dynamics of the system, which is described by a model of isothermal transient gas flow through pipelines.

Before constructing the model of natural gas systems, it is necessary to define the basic elements of such systems, namely nodes, pipes and compressors. Nodes are defined as the points where a node-connecting element (NCE; pipe or compressor) ends or where two or more NCEs join. Nodes are in fact delivery points, sources or just junctions. This study focuses on developing a systems science approach using optimal control theory for transient gas flow through pipelines. Therefore a source is considered as the discharge of a compressor which can supply gas at any desired pressure; it can also satisfy any flow demand. Since in this study it is assumed that no NCE (neither pipe nor compressor) exists ahead of a compressor, an explicit mathematical model for compressors is unnecessary. The inlet boundary condition for pipe flow is then the pressure equivalent to the discharge pressure of a compressor if one existed. The delivery point is characterized by the time-varying demand flow, i.e. the imposed outlet boundary condition. This flow must be supplied at around a specified contract pressure.

The pipelines dominate the major dynamic characteristics of the system. The mathematical model of gas flow through pipelines is described by partial differential equations (PDEs) based upon the principles of conservation of mass and momentum, the equation of state, together with a relationship accounting for the deviation of the gas from ideal gas behaviour. To construct this model, it is assumed that the flow is isothermal, unidirectional and turbulent. Model development and solution techniques for this problem have been extensively studied for more than 30 years. Most authors agree on describing the dynamics of the system by

$$\frac{B^2}{Ag_c}\frac{\partial m}{\partial x}+\frac{\partial P}{\partial t}=0, \tag{1}$$

$$\frac{\partial P}{\partial x}+\frac{1}{Ag_c}\frac{\partial m}{\partial t}+\frac{fm|m|B^2}{2DA^2g_c^2P}=0, \tag{2}$$

which are the equations of continuity and motion for transient gas flow respectively.

The derivation of these fundamental equations is given in Reference 1. These hyperbolic PDEs are dependent on space x and time t and are non-linear. They require, for the problem to be well posed, one boundary condition to be defined at each end of a pipeline for the dependent variables, pressure P and mass flow rate m, and one initial condition.

Equations (1) and (2) cannot be solved analytically because they are non-linear with respect to P and m. An analytical solution must incorporate some simplification or assume some specific set of initial and boundary conditions. Thus the equations for transient flow must be solved by approximate techniques.[1]

Several methods to solve the transient gas flow problem for pipelines have appeared in the literature. Taylor *et al.*[2] and Streeter and Wylie[3] have proposed the method of characteristics. Implicit finite difference procedures have been formulated by Guy,[4] Streeter and Wylie[3] and Wylie *et al.*[5] Rachford and Dupont[6] have presented a variational method using a Galerkin approximation technique. By further simplifications and modifications Osiadacz[7] has constructed a simpler diffusion-type PDE (i.e. a second-order parabolic PDE which is linear with respect to P^2); then he applies the finite difference method to solve the following resultant equation numerically:

$$\frac{\partial P^2}{\partial t}=\frac{DAB^2}{4fq}\frac{\partial^2 p^2}{\partial x^2}. \tag{3}$$

One of the earliest works on optimal control of gas pipelines is the study by Batey *et al.*[8] It seeks a reasonable control policy. The authors state some key rules for operation of the system with low energy consumption. Wong and Larson[9] have used dynamic programming to solve small problems such as a single compressor driving a single pipeline. Sood *et al.*[10] have presented a different approach for a similar system. In their study the dynamics of gas flow is given by the set of equations

$$\frac{A}{B^2}\frac{\partial \rho}{\partial t} = -\frac{\partial m}{\partial x}, \tag{4}$$

$$\frac{\partial \rho}{\partial x} = -\frac{2f\rho v^2}{D}. \tag{5}$$

Sood *et al.* have converted the original problem to one in which four simultaneous ordinary differential equations are to be solved. These equations have been solved using a Runge–Kutta method. For the minimization of the objective function which describes the energy consumption of a compressor, a gradient search method is employed. A hierarchical algorithm for the control of transient flow in a large, complex pipeline system has been described by Larson and Wismer.[11] In that study the network is decomposed into subsystems, with the policy in each subsystem being determined by means of the Wong and Larson[9] scheme. Hierarchical systems theory is then used to co-ordinate these individual subsystem solutions to achieve control of the overall network. At the upper level they use a heuristic that fixes the suction pressure. A non-linear programming algorithm has been described by Osiadacz and Bell[12] for this problem as well. This algorithm minimizes at each time step the fuel consumption of the gas engines which drive the reciprocating compressors. Osiadacz and Bell[13] have described a simplified algorithm for optimization of a large-scale gas network. The goal of their study is to minimize the fuel consumption of the gas engines at each compressor station. The pipeline dynamics is defined by equation (3). As they state, this formulation forces them to employ a volumetric flow rate which is averaged over the whole length of pipeline in every time intervals. In this method, at the first stage the pressures at all junctions and off-takes are calculated. Using the Crank–Nicolson procedure, the linearized equation is solved to compute the pressures along pipes. Next the flow through each compressor station is evaluated. Knowing the flow, suction and discharge pressure for each compressor, the objective function is minimized. After computing (if the algorithm intervenes) the optimal working parameters, the algorithm immediately moves to the next time level. It is clear that this approach is in fact a real time simulation.

In 1988 Marqués and Morari[14] presented a quadratic programming optimizer built around a dynamic simulator. It is a general on-line optimization scheme for operation of a gas pipeline network. They apply a special case of the 'control vector parametrization' method called the 'black box' technique. The values of the gradient of the objective function and the gradient of each constraint are obtained through simulations where each component of the *NP* (number of time steps in the operation horizon) control vector is perturbed. As a result, the computation of the gradients requires $m \sum k$, $k = 1, \ldots, NP$, simulations where m is the dimension of the control vector. It is obviously unfeasible in the matter of computation time and not necessarily exact.

Among all the above works, none of them has attempted to tackle the control of a gas pipeline under the transient flow condition by exploiting analytical tools of variational calculus without further assumptions rather than the ones used before. In this study the mathematical model, i.e. the set of non-linear PDEs (equations (1) and (2)) describing isothermal and unidirectional gas flow through the pipeline, is treated as it is. These equations are considered as the state equations of the optimal control problem to be constructed in the following section. The necessary conditions on the state and control variables to minimize the objective function(s) are obtained by applying the principles of calculus of variations directly to the infinite-dimensional model without any discretizations. They yield an optimal control policy which is the inlet pressure of the pipeline, i.e. the discharge pressure of the compressor.

The developed approach is examined in terms of controlling a pipeline subject to a sudden stepwise demand change. Three performance indices are applied to bring out the most suitable one. The optimal control policies for each case are presented.

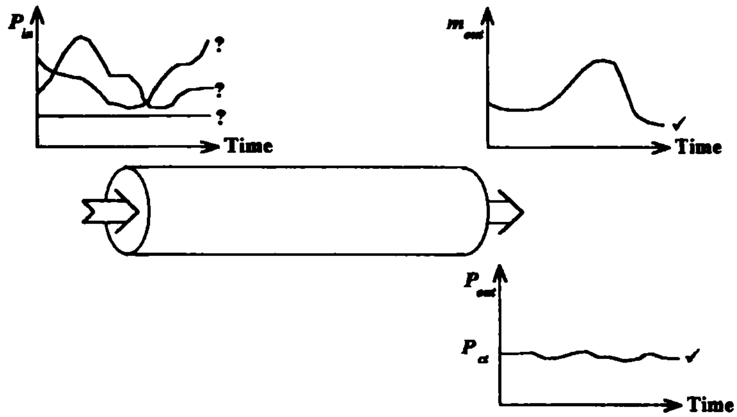

Figure 1. Schematic diagram of optimal control problem

2. MATHEMATICAL MODEL

In this optimal control study the dynamic simulator TRNFLOW is used. This simulator was developed by Rashidi *et al.*[15] and is built around the model presented by Wylie *et al.*[5]

Figure 1 summarizes the problem briefly: what should be the control policy for the inlet pressure P_{in} so that the outlet pressure P_{out} is as close as possible to the contract pressure P_{ct} and the time-varying demand m_{out} is satisfied?

State equations and performance indices

The governing equations of transient gas flow through pipes in plainer form are

$$\frac{\partial P}{\partial t} + a\frac{\partial m}{\partial x} = 0, \tag{6}$$

$$\frac{\partial m}{\partial t} + b\frac{\partial P}{\partial x} + c\frac{m|m|}{P} = 0, \tag{7}$$

where $m = m(x, t)$ is the mass flow rate and $P = P(x, t)$ is the gas pressure.

Initially it is assumed that the flow is in a steady state condition. Therefore mass flow rate at time zero, $m(x, 0)$, is constant and known, m_0. In addition, the initial pressure distribution $P(x, 0)$ can be calculated using steady state flow relationships. Summarizing,

$$m(x, 0) = m_0, \tag{8}$$

$$P(x, 0) = P_0(x). \tag{9}$$

The structure of the problem dictates the following boundary conditions:

$$m(L, t) = \theta(t), \quad t \geqslant 0, \tag{10}$$

$$P(0, t) = \beta(t), \quad t \geqslant 0, \tag{11}$$

where θ is the time-varying demand, assumed to be known, and β is the control input.

In this study three different performance indices are used to investigate their effects on the solution. These are

$$\text{Form1: } J = \frac{\omega}{2}\int_0^T [\min(P(L,t) - P_{\mathrm{ct}}, 0)]^2 \,\mathrm{d}t + \frac{\gamma}{2}\int_0^T \beta^2(t)\,\mathrm{d}t, \tag{12}$$

$$\text{Form2: } J = \frac{\omega}{2}\int_0^T [P(L,t) - P_{\mathrm{ct}}]^2 \,\mathrm{d}t + \frac{\gamma}{2}\int_0^T \beta^2(t)\,\mathrm{d}t, \tag{13}$$

$$\text{Form3: } J = \frac{\omega_1}{2}\int_0^T [\min(P(L,t) - P_{\mathrm{ct}}, 0)]^2 \,\mathrm{d}t + \frac{\omega_2}{2}\int_0^T [P(L,t) - P_{\mathrm{ct}}]^2 \,\mathrm{d}t + \frac{\gamma}{2}\int_0^T \beta^2(t)\,\mathrm{d}t. \tag{14}$$

All forms of objective functionals are defined so that at the extremum the outlet pressure will be as close as possible to and greater than the contact pressure (in Form1 and Form3 only) and will not allow the use of excessive control. In the above equations, ω and γ are used to balance the weights of the criteria of control and outlet pressure on the functionals, T is the final time and L is the length of pipeline. Note that Form3 is obtained by combining Form1 and Form2 to observe the transition between the two forms.

Optimal control problem

One of the equations (12)–(14) is used as an objective function of the optimal control problem. The state equations describe the dynamics of the system and the imposed initial and boundary conditions along with the bounds on the control variable constitute the constraints. As a result, the investigated optimal control problem can be briefly represented as

$$\begin{aligned} &\text{minimize} && J \\ &\text{subject to} && \text{equations (6) and (7)} \\ & && \text{initial and boundary conditions} \\ & && \beta \in K \end{aligned} \tag{15}$$

where $K = \{\beta(t)\colon \beta_{\min} \leqslant \beta(t) \leqslant \beta_{\max}\}$.

Equations characterizing optimal variables

In classical optimal control theory, such constrained problems are handled by the Lagrange multiplier approach.[16] In this method the constraints (i.e. state equations) are incorporated into an objective functional to formulate an augmented objective functional through the use of Lagrange multipliers. In other words, the inner products of state equations with adjoint states (Lagrange multipliers) are added to the original objective functional to get an unconstrained optimal control problem (except for constraints on the control variables). In this study the augmented objective functional J_{a} defined by introducing Lagrange multipliers λ and μ. For example, for Form1,

$$\begin{aligned} J_{\mathrm{a}} = {} & \frac{\omega}{2}\int_0^T [\min(P(L,t) - P_{\mathrm{ct}}, 0)]^2 \,\mathrm{d}t + \frac{\gamma}{2}\int_0^T \beta^2(t)\,\mathrm{d}t + \int_0^T\!\!\int_0^L \lambda\left(\frac{\partial P}{\partial t} + a\frac{\partial m}{\partial x}\right)\mathrm{d}x\,\mathrm{d}t \\ & + \int_0^T\!\!\int_0^L \mu\left(\frac{\partial m}{\partial t} + b\frac{\partial P}{\partial x} + c\frac{m|m|}{P}\right)\mathrm{d}x\,\mathrm{d}t. \end{aligned} \tag{16}$$

Obviously, by equating the gradients of J_{a} with respect to (w.r.t.) λ and μ to zero, one may deduce the state equations (6) and (7).

Owing to the initial and boundary conditions on pressure, the admissible perturbations η of pressure (i.e. $P \to P + \varepsilon\eta$, where ε is an arbitrary positive number) are set to zero for all x at $t = 0$ and for all t at $x = 0$ and are set free for all x at $t = T$ and for all t at $x = L$. By admissible η we mean that $P + \varepsilon\eta$ must satisfy the same conditions as P does (i.e. $(P + \varepsilon\eta)(x, 0) = P_0(x)$ and $(P + \varepsilon\eta)(0, t) = \beta(t)$. The derivative of J_a w.r.t. P in the direction of η yields

$$D_P(P, m, \beta)(\eta, 0, 0) = \int_0^L \left(\lambda\eta\Big|_0^T - \int_0^T \frac{\partial\lambda}{\partial t}\eta \, dt \right) dx + \int_0^T \left[b\mu\eta\Big|_0^L - \int_0^L \left(b\frac{\partial\mu}{\partial x}\eta + c\mu\frac{m|m|}{P^2}\eta \right) dx \right] dt$$
$$+ \omega \int_0^T \int_0^L \operatorname{sign}[\min(P(x, t) - P_{ct}, 0)](P(x, t) - P_{ct})\delta(x - L^-) \, dx \, dt. \qquad (17)$$

Then the gradient becomes

$$\begin{aligned} \nabla_P J_a(P, m, \beta) &= -\frac{\partial\lambda}{\partial t} - b\frac{\partial\mu}{\partial x} - c\mu\frac{m|m|}{P^2}, \quad 0 < t < T, \quad 0 < x < L \\ &= \lambda(x, T), \quad 0 < x < L \\ &= b\mu(L, t) + \omega \operatorname{sign}[\min(P(L, t) - P_{ct}, 0)](P(L, t) - P_{ct}), \quad 0 < t < T. \end{aligned} \qquad (18)$$

Since this problem is unconstrained w.r.t. P, we may obtain one of the adjoint state equations by equating this gradient to zero. Thus

$$\frac{\partial\lambda}{\partial t} + \frac{\partial\mu}{\partial x} + c\mu\frac{m|m|}{P^2} = 0, \quad 0 < t < T, \quad 0 < x < L, \qquad (19)$$

$$\lambda(x, T) = 0, \quad 0 < x < L, \qquad (20)$$

$$\mu(L, t) = -\frac{\omega}{b}\operatorname{sign}[\min(P(L, t) - P_{ct}, 0)](P(L, t) - P_{ct}), \quad 0 < t < T. \qquad (21)$$

Similarly, for the other dependent variable m we get the second adjoint state equation

$$a\frac{\partial\lambda}{\partial x} + \frac{\partial\mu}{\partial t} - \frac{2c\mu m}{P^2}\operatorname{sign}(m) = 0, \quad 0 < t < T, \quad 0 < x < L, \qquad (22)$$

$$\mu(x, T) = 0, \quad 0 < x < L, \qquad (23)$$

$$\lambda(0, t) = 0, \quad 0 < t < T. \qquad (24)$$

Equations (20) and (23) are initial conditions for λ and μ respectively. It may have been already noticed that the adjoint state equations are known at the final time (i.e. they are set to zero for $t = T$) while the state equations are known at the initial time. Therefore the adjoint state equations are solved backwards in time. Equations (21) and (24) specify the boundary conditions for the adjoint state variables in such a way that one adjoint state variable is fixed at each end of the pipeline.

The problem is constrained w.r.t. the control variable β (i.e. the controls are assumed to lie in a convex set K). Pontryagin's minimum principle[16] gives a variational inequality for the optimum control function $\beta^*(t)$:

$$\int_0^T \nabla_\beta J_a(P, m, \beta)(\beta - \beta^*) \, dt \geqslant 0 \quad \forall\beta \in K. \qquad (25)$$

It is necessary to modify the cost function in order to obtain $\nabla_\beta J_a(P, m, \beta)$. The modified but equivalent cost function J_{ma} is given as

$$
\begin{aligned}
J_{ma} = & \frac{\omega}{2}\int_0^T [\min(P(L,t) - P_{ct}, 0)]^2 \, dt + \frac{\gamma}{2}\int_0^T \beta^2(t)\, dt + \int_0^T \int_0^L \lambda\left(\frac{\partial P}{\partial t} + a\frac{\partial m}{\partial x}\right) dx\, dt \\
& + \int_0^T \int_0^L \mu\left(\frac{\partial m}{\partial t} + c\frac{m|m|}{P}\right) dx\, dt + \int_0^T \left(b\mu P\Big|_0^L - \int_0^L bP\frac{\partial \mu}{\partial x} dx\right) dt.
\end{aligned} \tag{26}
$$

Integration of the last term yields

$$
J_{ma} = \cdots + \frac{\gamma}{2}\int_0^T \beta^2(t)\, dt + \cdots + \int_0^T \left(b\mu(L,t)P(L,t) - b\mu(0,t)\beta(t) - \int_0^L bP\frac{\partial \mu}{\partial x} dx\right) dt. \tag{27}
$$

Note that $\beta(t) = P(0, t)$ and $J_a = J_{ma}$.

The directional derivative of the modified cost function J_{ma} becomes

$$
D_\beta(P, m, \beta)(0, 0, \eta) = \int_0^T [\gamma\beta(t)\eta - b\mu(0,t)\eta]\, dt. \tag{28}
$$

As a result the gradient is

$$
\nabla_\beta J_a(P, m, \beta) = \gamma\beta(t) - b\mu(0, t). \tag{29}
$$

3. NUMERICAL DETERMINATION OF OPTIMAL CONTROL

The variational approach summarized above leads to two-point non-linear initial boundary value problems that cannot be solved analytically to obtain the optimum control law. In the literature a number of numerical techniques (e.g. steepest descent, variation of extremals, quasi-linearization) for determining optimal controls and trajectories are available. In this study the method of steepest descent[16] (or gradients) is used for finding the optimal control of the pipeline problem.

The algorithmic procedure we use to solve the optimal control of the pipe flow problem by the steepest descent method can be described in the following steps.

1. Select a discrete approximation to the control policy $\beta(t)$, $0 < t < T$. Let the iteration index $k = 0$.
2. Using the control policy, solve the state equations (apply the simulator TRNFLOW) from zero to T and store the resulting trajectories $P(x, t)$ and $m(x, t)$.
3. With the known control and state variables, solve the adjoint state equations from T to zero (backwards in time).
4. If the stopping criterion

$$
\|\nabla_\beta J\| < \varepsilon
$$

(where ε is a predetermined positive constant) is not satisfied, generate a new control function

$$
\tilde{\beta}_{k+1}(t) = \beta_k(t) - \alpha\nabla_\beta J_a(P, m, \beta). \tag{30}
$$

Note that since the control is constrained, one cannot expect $\|\nabla_\beta J\|$ to approach zero as β_k goes to the optimum β. Instead, several alternative stopping criteria can be utilized simultaneously. For instance, the algorithm may stop if a specified number of iterations is exceeded or $\|\beta_{k+1}(t) - \beta_k(t)\|^2$ is sufficiently small. Furthermore, the function $\tilde{\beta}_{k+1}(t)$ generated above is

Table I. Thermodynamic data of flowing gas

Ambient temperature	60°F
Gas gravity	0·55
Pseudocritical pressure	672 psi(absolute)
Pseudocritical temperature	345 °R

not necessarily a member of K. Hence the actual $\beta_{k+1}(t)$ should be obtained by taking the projection of $\tilde{\beta}_{k+1}(t)$ onto the set K as

$$\beta_{k+1}(t) = \begin{cases} \beta_{\min}, & \tilde{\beta}_{k+1}(t) < \beta_{\min}, \\ \beta_{\max}, & \tilde{\beta}_{k+1}(t) > \beta_{\max}, \\ \tilde{\beta}_{k+1}(t), & \text{otherwise.} \end{cases} \tag{31}$$

5. Replace $\beta_k(t)$ by $\beta_{k+1}(t)$ and return to step 2.

The step size α is determined by some one-dimensional search techniques. The value used for the termination constant ε is determined arbitrarily.

4. RESULTS AND DISCUSSION

The developed approach has been applied to a gas pipeline. The pipeline has a length of 2×10^5 ft and a flow diameter of 2 ft. The friction factor is constant (0·015) throughout the pipeline. The thermodynamic data of the flowing gas are given in Table I.

In this example the system is subject to the demand history depicted in Figure 2. The outlet pressure should not be dropped below the contract pressure of 500 psi. It is known that after 2·5 h, demand will suddenly increase by one-third. This will last 1 h and the flow will then return to its former state. If the pipeline is controlled by a constant inlet pressure of 700 psi, the outlet pressure will decrease and then start to increase after the period of high demand (see Figure 3). As seen in Figure 3, the pipeline is pressurized more than enough except for the duration of high demand; nevertheless, the outlet pressure exceeds the lower limit of pressure for this period of time, which means clearly more energy consumption than necessary.

The inlet pressure policies shown in Figures 4–11 are obtained by the algorithm described above. Control histories are denoted by full curves. These figures also show the pressures at the delivery points (outlet pressures) by dotted curves.

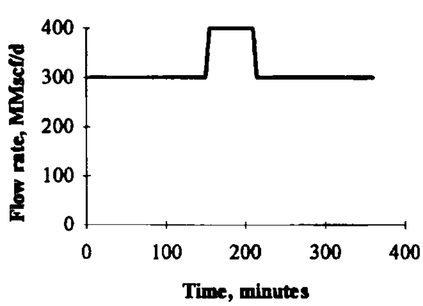

Figure 2. Demand history at outlet

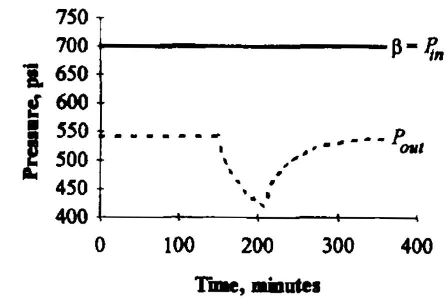

Figure 3. Constant control (inlet pressure) and outlet pressure

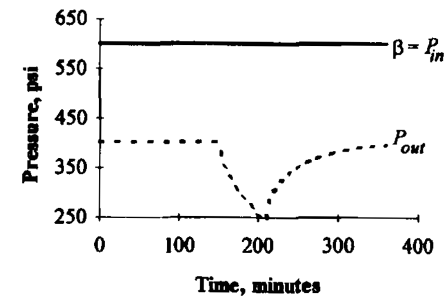

Figure 4. Control obtained by using Form1 ($\gamma = 10^{-4}$, $\omega = 1$), initial guess $P(0, t) = 600$ psi, and outlet pressure

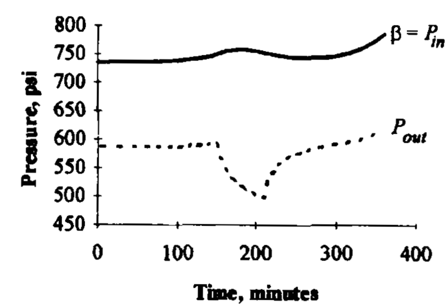

Figure 5. Control obtained by using Form1 ($\gamma = 10^{-4}$, $\omega = 1$), initial guess $P(0, t) = 800$ psi, and outlet pressure

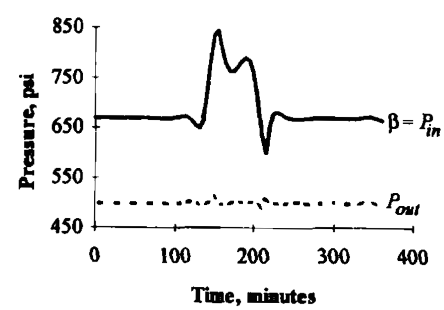

Figure 6. Control obtained by using Form2 ($\gamma = 10^{-4}$, $\omega = 1$), initial guess $P(0, t) = 600$ psi, and outlet pressure

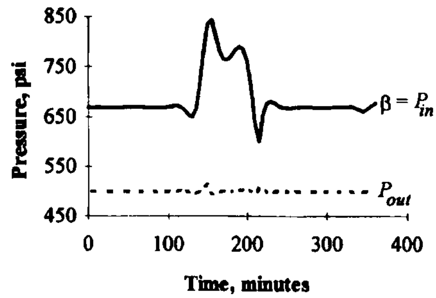

Figure 7. Control obtained by using Form2 ($\gamma = 10^{-4}$, $\omega = 1$), initial guess $P(0, t) = 800$ psi, and outlet pressure

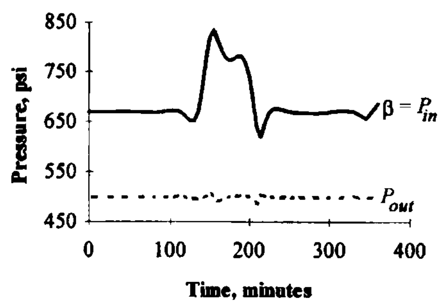

Figure 8. Control obtained by using Form3 ($\gamma = 10^{-4}$, $\omega_1 = 0{\cdot}25$, $\omega_2 = 0{\cdot}75$), initial guess $P(0, t) = 800$ psi, and outlet pressure

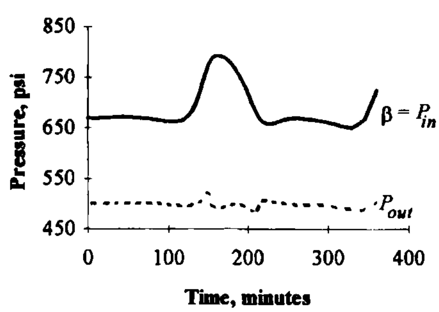

Figure 9. Control obtained by using Form3 ($\gamma = 10^{-4}$, $\omega_1 = 0{\cdot}40$, $\omega_2 = 0{\cdot}60$), initial guess $P(0, t) = 800$ psi, and outlet pressure

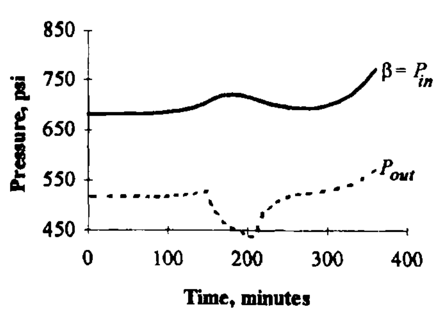

Figure 10. Control obtained by using Form3 ($\gamma = 10^{-4}$, $\omega_1 = 0{\cdot}70$, $\omega_2 = 0{\cdot}30$), initial guess $P(0, t) = 800$ psi, and outlet pressure

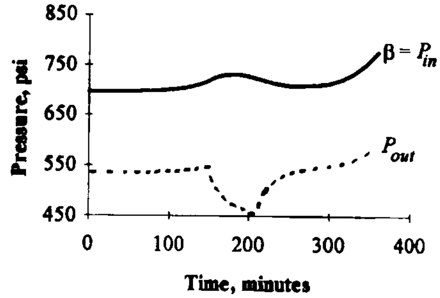

Figure 11. Control obtained by using Form3 ($\gamma = 10^{-4}$, $\omega_1 = 0{\cdot}90$, $\omega_2 = 0{\cdot}10$), initial guess $P(0, t) = 800$ psi, and outlet pressure

Table II. Initial and final costs for two different initial cases

	Initial control $P(0, t) = 600$ psi		Initial control $P(0, t) = 800$ psi	
	Initial	Final	Initial	Final
Form1	$2{\cdot}04 \times 10^8$	$2{\cdot}04 \times 10^8$	$6{\cdot}91 \times 10^5$	$6{\cdot}05 \times 10^5$
Form2	$2{\cdot}04 \times 10^8$	$5{\cdot}74 \times 10^5$	$2{\cdot}40 \times 10^8$	$5{\cdot}75 \times 10^5$

Table III. Initial and final costs for four different combinations of weights ω_1 and ω_2

	$\omega_1 = 0{\cdot}25$, $\omega_2 = 0{\cdot}75$		$\omega_1 = 0{\cdot}40$, $\omega_2 = 0{\cdot}60$		$\omega_1 = 0{\cdot}70$, $\omega_2 = 0{\cdot}30$		$\omega_1 = 0{\cdot}95$, $\omega_2 = 0{\cdot}05$	
	Initial	Final	Initial	Final	Initial	Final	Initial	Final
Form1	$1{\cdot}81 \times 10^8$	$5{\cdot}94 \times 10^5$	$1{\cdot}45 \times 10^8$	$8{\cdot}68 \times 10^5$	$7{\cdot}26 \times 10^7$	$6{\cdot}86 \times 10^6$	$2{\cdot}47 \times 10^7$	$4{\cdot}01 \times 10^6$

The results of this model must be considered with respect to the parameters γ and ω (or ω_1 and ω_2) and the form of the cost functions. If any or all of these parameters and the form of the cost functions change, then different results will of course be obtained.

For the cost function types of Form1 and Form2 the algorithm is initiated from two different initial guesses, namely $P(0, t) = 600$ psi and $P(0, t) = 800$ psi for $0 < t < T$. Using Form1, the algorithm does not progress if the initial guess is $P(0, t) = 600$ psi (Figure 4). Using the higher pressure (800 psi) as an initial starting point yields a better result (Figure 5). These results reveal that the model employing the cost function with the 'min' term fails. On the other hand, the model with Form2 provides the best optimal control regardless of the initial constant control (see Figures 6 and 7). These two figures demonstrate the superior performance of this control in keeping the outlet pressure around the contract pressure. Initial and final values of the performance index are given in Table II for each case. Consequently, the results give an idea about which form is suitable.

Form3 throws light on this situation. It is a combination of the previous two forms. The weight of terms with and without 'min' is adjusted by ω_1 and ω_2 respectively. For this case the results are presented in Figures 8–11 and Table III. As can be understood from these results, decreasing the weight of the term with 'min' (ω_1) creates an improvement in the final solution.

5. CONCLUSIONS

A general optimal control approach has been proposed for the control of a gas pipeline. The developed model tries to find the optimal control policy while keeping the system in a desired state.

The present work has the main advantage that the descent direction in which the search for the minimum progresses is obtained directly by an analytical approach. It is certainly superior to any numerical approach. Therefore the computation time is shorter than the other methods.

Investigations into the effect of cost function type show the suitability of an *equality* constraint on the outlet pressure rather than a *greater than or equality* constraint.

Having the time constant characteristic, the control is devised so that future events (peaks in demand) are taken into account.

Work is in progress to deal with the control of gas networks.

APPENDIX: NOMENCLATURE

a, b, c	constants
A	cross-sectional area of pipe
B	isothermal speed of sound in gas
D	diameter of pipe
f	friction factor
g_c	conversion factor for gravity
J	performance index
K	set of admissible control
L	length of pipe
m	mass flow rate
P	gas pressure
q	volumetric flow rate
t	time
T	final time
v	average velocity of gas
x	distance co-ordinate

Greek letters

α	step size
β	control function
γ, ω	weight factors
ε	a positive number, convergence parameter
η	admissible perturbation
θ	function representing time-varying demand
λ, μ	Lagrange multipliers
ρ	density of gas
δ	Dirac Delta function

Subscripts

a	augmented
ct	contract
in	inlet
k	iteration index

ma	modified augmented
min	minimum
max	maximum
out	outlet

Superscripts

*	optimal value
–	limit from below

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